Multirate coupling of controlled rectifier and non-linear finite element model based on Waveform Relaxation Method

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To study a multirate system, each subsystem can be solved by a dedidated sofware with respect to the physical problem and the time constant. Then, the problem is the coupling of the solutions of the subsystems. The Waveform Relaxation Method (WRM) seems to be an interesting solution for the coupling but it has been mainly applied on academic examples. In this communication, the WRM method has been applied to perform the coupling of a controlled rectifier and a transformer modeled by a non-linear finite element model.

***Index Terms*—Waveform relaxation method, multirate system, finite element method.**

I. Introduction

ELECTRICAL system can involve several devices which have different physics and dynamics. Each device can be represented by a numerical model. To study the system, the coupling of the numerical models is necessary. A strong coupling of the subsystems involves a step time according to the smallest time constant, and thus a long computation time. Another approach is to use a weak coupling of the numerical models. In this case, a dedicated sofware can be used for each device with respect to its own time constant. To ensure a consistent coupling, the waveform relaxation method (WRM) can be investigated to solve this kind of problem [1], [2]. The WRM approach is an iterative process which converges in theory to the solution of a strong coupling [3]. It should be notice that this approach has been until now mainly applied to study academic example.

In this communication, the WRM approach is applied to study a realistic system involving its control loop, a single phase transformer associated with a controlled rectifier. The transformer is modeled by a non-linear finite element model and the rectifier is controlled by a pulse width modulation (PWM) technique. Each subsystem is studied with a time step adapted to its time constant [4].

II. Waveform relaxation method

Let consider a device composed of r subsystems, each subsystem i satisfying

$$\dot{\mathbf{y}}_i(t) = \mathbf{f}_i(\mathbf{y}(t), \mathbf{z}(t)) \quad (1)$$
$$0 = \mathbf{g}_i(\mathbf{y}(t), \mathbf{z}(t)), \quad (2)$$

with $t \in [0, T]$ and the initial conditions $\mathbf{y}(0) = \mathbf{y}_0$ and $\mathbf{z}(0) = \mathbf{z}_0$, $\mathbf{y}$ being the differential variables and $\mathbf{z}$ the algebraic variables. The WRM computes iteratively an approximation $\left(\tilde{\mathbf{y}}^k(t), \tilde{\mathbf{z}}^k(t)\right)$ of the exact solution. The first step is the extrapolation step: for $k = 0,\ \tilde{\mathbf{y}}^k(t) = \mathbf{y}_0,\ \tilde{\mathbf{z}}^k(t) = \mathbf{z}_0,\ \forall t \in [0, T]$. Then at the iteration k and for the subsystem i, the algorithm solves

$$\dot{\mathbf{y}}_i(t) = \mathbf{f}_i(\mathbf{Y}_i^k(t), \mathbf{Z}_i^k(t)) \quad (3)$$
$$0 = \mathbf{g}_i(\mathbf{Y}_i^k(t), \mathbf{Z}_i^k(t)). \quad (4)$$

The value of $\mathbf{Y}_i^k$ (resp. $\mathbf{Z}_i^k$) depends on $\tilde{\mathbf{y}}^{k-1}$ and $\tilde{\mathbf{y}}^k$ (resp. $\tilde{\mathbf{z}}^{k-1}$ and $\tilde{\mathbf{z}}^k$) and on the relaxation schemes (Picard, Jacobi or Gauss-Seidel). For example, with the Gauss-Seidel scheme, the subsystems are solved sequentially with

$$\mathbf{Y}_i^k(t) = [\tilde{\mathbf{y}}_1^k, \ldots, \tilde{\mathbf{y}}_{i-1}^k, \tilde{\mathbf{y}}_i^k, \tilde{\mathbf{y}}_{i+1}^{k-1}, \ldots, \tilde{\mathbf{y}}_r^{k-1}]^{\mathrm{T}}, \quad (5)$$

and

$$\mathbf{Z}_i^k(t) = [\tilde{\mathbf{z}}_1^k, \ldots, \tilde{\mathbf{z}}_{i-1}^k, \tilde{\mathbf{z}}_i^k, \tilde{\mathbf{z}}_{i+1}^{k-1}, \ldots, \tilde{\mathbf{z}}_r^{k-1}]^{\mathrm{T}}. \quad (6)$$

Subsystems are solved on the overall time domain $[0, T]$, then the waveforms are transfered from one subsystem to the others. Each subsystem is solved with respect to its own time step. Since the waveforms $\mathbf{y}_i$ and $\mathbf{z}_i$ of each subsystem are sampled with different time steps, interpolation technic enables to transfer the waveforms from one subsystem to another.

III. Application

Let us consider a transformer and its associated rectifier. The rectifier is controlled to provide a direct voltage v_c of 800 V and a current i_s into the secondary winding in phase with the nominal voltage v_{20}; moreover, a current i_{ch} is imposed into the rectifier circuit as a load current. The control based on a PWM requires a very small time step Δt_r. The transformer is modeled by a finite element (FE) method with a magnetic vector potential formulation. The voltages v_{10} and v_s are respectively imposed to the primary and secondary windings. The solution of the FE model using the time step Δt_r is unfeasible in terms of computation time. The WRM allows to use a bigger time step allowing to dramatically reduce the computation time.

The WRM is applied to perform the coupling between the FE model of the transformer and the circuit model of the rectifier. A classic source coupling [5] does not allow to control the current i_s into the secondary winding. Therefore, a coupling parameter is used [6], [7]. A resistor R and an inductance L are introduced into the rectifier circuit (Fig.1) to represent a behaviour close to the transformer one. The resistance is computed according to the resistivity and the length of the winding, and the inductance by a calculation based on a linear FE model of the transformer. Then a residual current source i_{res} is added in parallel (Fig. 1) to guarantee the consistency of the coupling.

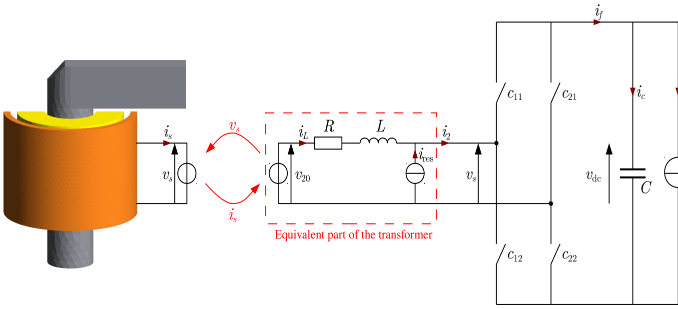

Fig. 1. WRM split device

At each iteration k of the WRM process, the source of the FE model is the voltage v_s^k, and its solution provides the current i_s^k in the secondary winding. The voltage v_s^k is given by the solution of the rectifier model. In this model, the residual current i_{res}^k is a source; the voltage v_{dc}^k is controlled to be equal to 800 V and the current i_2^k is also controlled to be in phase with the nominal voltage v_{20}. The residual current is such that $i_{\text{res}}^k = i_s^{k-1} - i_L^{k-1}$. In this manner, over the iterations of the WRM process, the current i_s^k converges to i_2^k. At the end of the process, i_s^k is in phase with the nominal voltage v_{20}. Fig. 2 presents the convergence criterion related to the current i_s of the WRM process: the convergence is effective after 4 iterations. According to the parameter coupling, the current i_s^k tends to be in phase with the nominal voltage v_{20} (Fig. 3). Furthermore, the voltage v_{dc} is close to 800 V. Fig. 4 shows the current i_p into the primary winding for linear or non-linear FE model. In the non-linear model, the reluctivity of the magnetic core depends on the magnetic field.

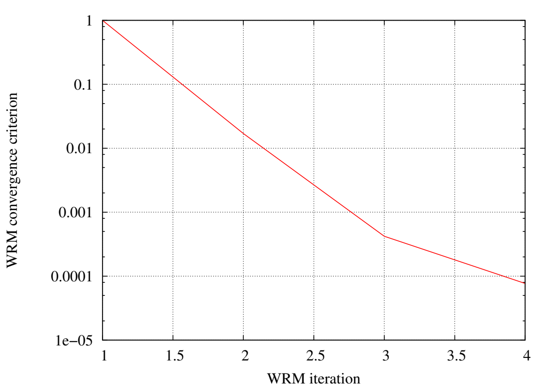

Fig. 2. Convergence criterion of the WRM iteration.

The waveform relaxation method is well-adapted to the coupling and the simulation of multirate systems. It allows

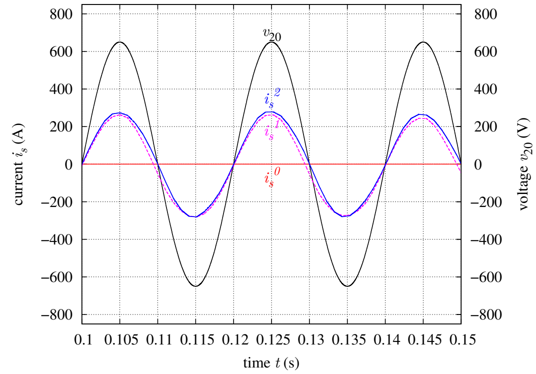

Fig. 3. Current i_s^k into the secondary winding with respect to the WRM iteration and nominal voltage v_{20}.

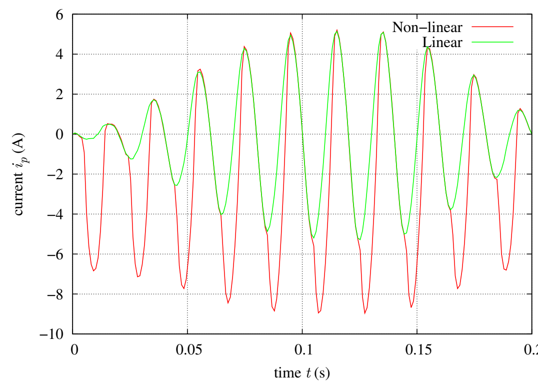

Fig. 4. Current i_p into the primary winding for linear and non-linear finite element model.

to use a time discretisation per subsystem. Considering the parameter coupling, the output current of a finite element model can be controlled whereas this model is not into the controlled circuit model. Moreover, the finite element model can be linear as well as non linear without changing into the coupling.

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